7.3 Double Angle Formulas

A Double-Angle Identities

The double-angle identities are:

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Ex 1. Given that $\sin x = -\frac{1}{3}$ and that the angle x is in the third quadrant, find $\sin 2x$, $\cos 2x$ and $\tan 2x$.

B Alternate Formulas

The double angle cosine has the following alternate formulas:

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = 2\cos^2 x - 1$$
$$\cos 2x = 1 - 2\sin^2 x$$

Ex 2. Use the double angle formulas and the fundamental trigonometric identity $\sin^2 x + \cos^2 x = 1$ to prove the alternate formulas.

C Half-Angle Identities

The half-angle identities are:

$$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$$
$$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$$
$$\tan^2 \frac{x}{2} = \frac{1-\cos x}{1+\cos x}$$

or:

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$
$$\cos \frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$
$$\tan \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

Ex 3. Find the exact value of:

a) $\sin \frac{\pi}{8}$

b) $\cos \frac{\pi}{12}$

c) $\tan \frac{5\pi}{8}$

D Triple-Angle Identities

The triple-angle identities are:

$$\sin 3x = 3\sin x - 4\sin^3 x$$
$$\cos 3x = 4\cos^3 x - 3\cos x$$
$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Ex 4. Prove the triple-angle identities.

E Half-Angle Tangent Formulas (Weierstrass *t*-substitution)

The following formulas are called half-angle tangent formulas or Weierstrass *t*-substitution:

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$
$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$
$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

Ex 5. Prove the half-angle tangent (Weierstrass *t*-substitution) formulas.

Reading: Nelson Textbook, Pages 402-406
Homework: Nelson Textbook, Page 407: #1abd, 2ae, 4, 5, 9, 10, 11, 12, 13, 15

